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Survivability and Recovery of Degraded Communication Networks

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***Abstract*—When multiple nodes in a network are subject to failure or loss, the question arises as to whether communication across the resulting degraded network is feasible. Percolation theory and random graph theory have been previously used to answer this question. Here we extend random geometric graph theory to the case of networks with some randomness in link formation. In addition, initial steps toward addressing the little studied question as to *how to recover* from failures that destroy network connectivity are shown here.**

I. Introduction

There has been increased interest in the survivability of networks when some of the nodes or links may be subject to failure. Such failures could occur because of accidents, malfunctions, natural or man-made disasters, or intentional attacks.

A number of works, such as [1], have applied percolation theory to address the question of survivability to random loss of links in wireless networks. Application of percolation theory for correlated cascading failures in a random geometric graph model was investigated in [2]. Other works have utilized percolation theory to study the survivability of the Internet under random failures [3] as well as under intentional attacks [4]. These works investigate the behavior near the critical point of quantities such as critical exponents and the size of the largest spanning cluster. In particular, the existence of a single giant component, or network spanning cluster, has been much studied. All of these previous studies assume that the links operate at the same constant data rate and transmission power before and after the failures; the only change to the network is that after failures, some links are removed.

In this paper we begin an examination of the question of how to adjust the network to recover from the destruction of connectivity after failures of randomly selected nodes or a power shortage. A key point in determining the survivability of a wireless communications network is that the connectivity of a network, or even of a pair of nodes, depends crucially on the link rate, or equivalently on the transmit power and the distance between nodes. This dependence has not been previously utilized *to overcome failures* in past studies of network reliability and survivability, which instead have primarily focused on characterising under which conditions communication can be established. Here, instead, a method is proposed to leverage this crucial dependence in order to *enable recovery of communications after a catastrophic failure.*

In different contexts some previous works have also considered this distance dependence. The allusion to the concept of rescaled graphs maintaining the same percolation properties in [5] is similar to the distance rescaling we propose following failures to maintain connectivity of all nodes. A distance rescaling is the first step of our proposed recovery mechanisms. In [6] adjustment of transmission power is proposed for nodes subject only to distance dependent path loss, so as to ensure a giant component in the network when there are local spatial inhomogeneities. We consider transmission power adjustment as one of several potential mechanisms to re-establish network connectivity following catastrophic failure and our proposed distance rescaling. One question we investigate is at what rate transmission of a single message in the network can be made after the removal of a subset of the network's nodes, if transmission power can not be increased. We also consider more general channel models than [6], in the context of random loss of bonds. We further relate these mechanisms to bond loss, and introduce a random link channel component into our rescaling. Finally, we show how spatial inhomogeneities in node density are related to bond percolation.

The idea proposed here is that after failure of multiple nodes or the occurrence of a power shortage destroys the connectivity across a network, the *definition of an edge between nodes should be revised.* In other words, if enough nodes fail in a network so that connectivity can no longer be achieved, we propose renormalization of the *criteria for a link forming.* We introduce several methods by which this renormalization of the criteria may be implemented in practice.

In the case of failure of multiple nodes destroying network connectivity, our recovery mechanism would make the criteria for forming a link less stringent. Therefore, links or edges are then allowed to form between nodes separated by greater distances than before the failures. Specifically, a distance threshold below which a link or edge is considered to be available should be rescaled to be larger. This larger distance threshold is chosen to enable the network to achieve connectivity. The failure of nodes and subsequent rescaling of distance, which yields the formation of new additional edges or links, results in the network looking similar in terms of degree distribution to the original network prior to the node failures. This proposed rescaling of distances and resulting formation of new links upon loss of network connectivity is similar in spirit, but different in technique and result, to the renormalization group methods of statistical physics.

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As a result of the proposed rescaling of the distance threshold, we can not only recover from loss of nodes, but also from other types of failures. We also consider the case in which the transmit power of the nodes needs to be reduced, due for example to Low Probability of Detection (LPD) constraints, energy conservation, or failure of power stations. If it becomes necessary for nodes to reduce power, the *criteria for forming an edge or link should become more stringent*, if the same original link rate is desired. In this case, the distance threshold below which two nodes can directly communicate is reduced. This downward distance rescaling to accommodate decreased power availability will break some links, but in many cases network connectivity may still be maintained.

The rest of this paper is organized as follows: In Section II we describe the network model. We propose a recovery mechanism and show the distance rescaling needed to maintain connectivity in Section III. In Section IV we propose several methods by which the necessary distance rescaling can be achieved. A summary is given in Section V.

II. Network and Modes of Failure Model

Here we begin with the model of random geometric graph theory [7], which handles the case of randomly placed nodes, where the existence of an edge, or in communications a link, is determined solely based on whether the distance between the nodes exceeds a specified threshold. This model serves as our starting point, since in numerous wireless communication networks the connectivity between any pair of nodes is strongly dependent on the distance between them. However, since the wireless channel often depends on additional factors that are modeled as random channel components, we also introduce a random bond component to our model in Section III-B, and determine criticality in terms of loss of bonds as well.

Following the random geometric graph model [7], we assume N nodes are randomly placed according to a Poisson process in a square of length L in d dimensions. Any two nodes are considered "connected" if the distance between the two nodes does not exceed a specified distance threshold r_N; otherwise they are not "connected". Edges are formed between all pairs of connected nodes. An edge indicates that the nodes can receive transmissions from each other in the absence of other transmissions in the network.

It is assumed that initially every node in the network can be reached by every other node through some path of edges. Then a fraction f of the nodes are randomly selected for removal. These removed nodes could have failed, for example, due to an accident, an intentional attack, or a power limitation. If the fraction f satisfies

$$f < 1 - p_c, \tag{1}$$

where the percolation threshold p_c is given by [3]

$$p_c = \frac{1}{K-1}, \tag{2}$$

where

$$K = \frac{< k^2 >}{< k >}, \tag{3}$$

then the nodes can still communicate with each other at their original data rate. In this case, the degree of a node is k, and it is seen that the degree distribution determines the percolation threshold. The percolation threshold (2) applies either to the fraction of nodes needed or the fraction of bonds needed for a single giant component to span the network.

However, if (1) is not satisfied, then many pairs of nodes in the network can not communicate with one another. In this case, we propose the following modification to the network so as to enable all nodes to communicate with one another: We increase r_N to $r_{N'}$, where $N' = N(1-f)$ is the number of nodes remaining in the network after the removal of the failed nodes. The new maximal edge distance $r_{N'}$ is derived in Section III-B to be the smallest distance such that all nodes left in the network can communicate with each other through a path across the new, potentially larger, edges. In a finite sized network, this scaling up of r_N can be thought of as scaling down the network length L, as it will now take fewer hops to traverse the network.

III. Renormalizing Distance after Catastrophic Failure

After loss or failure of a random sample of nodes, we propose rescaling the distance threshold so as to create new links or edges in order to preserve the statistical connectivity properties of the graph. In Section IV we discuss several methods to implement such distance rescaling. We first consider in Section III-A the random geometric graph model, in which the criteria for edge formation depends solely on the distance between nodes. Extensions to the case in which the presence of a link between two nodes also depends on a random component are discussed in Section III-B. Generalization to inhomogeneous networks is outlined in Section III-C. The relation between the results here and percolation theory is discussed in Section III-D.

A. Distance Dependent Channel

We start with a random geometric graph theory model, and derive how to re-establish connectivity when it is broken due to the failure of a number of nodes. In Section III-B, we generalize this model and our connectivity re-establishment method to the case in which there is also a random component to the formation of bonds.

In analogy to the discussion in [7], the distance r_N between two nodes in a fixed volume of size L^d, where L is the linear dimension of the d-dimension region, is

$$r_N \sim C^{1/d} \times \left(\frac{\ln N}{N}\right)^{1/d}. \tag{4}$$

The constant C is defined so that the expected degree of a point is asymptotically $C \ln N$ [7]. Hence, using the Poisson approximation for node distribution it is seen that [7] $C > 1$ is required for all nodes to be connected, and it is also sufficient as well.

We now consider the situation in which a fraction f of the N nodes are removed. Then we propose rescaling length so that the distance threshold becomes

$$r_{N'} = \frac{r_N}{(1-f)^{1/d}} \times \left(1 + \frac{\ln(1-f)}{\ln N}\right)^{1/d} \tag{5}$$

When $\mid \frac{1}{1-f} \mid << N$, equation (5) can be approximated by

$$r_{N'} \approx \frac{r_N}{(1-f)^{1/d}} \times \left(1 - \frac{\ln(\frac{1}{1-f})}{d \times \ln N}\right) \quad (6)$$

It is seen that loss of a larger fraction f of nodes necessitates a greater distance renormalization to maintain connectivity. For a fixed f, if the original network in a fixed volume has a smaller number of nodes N, then (6) shows that the rescaling factor will need to be slightly smaller than if the original N were larger. If losses are correlated spatially, then the network will need to renormalize to a larger distance scale, to maintain connectivity; the correlation has the same effect as amplifying f.

B. Generalized Channel

We now consider the situation in which there can be a combination of link and node failures. Equivalently, we handle the case in which whether a link forms between any two nodes depends not only on the distance between nodes, but also on a random component that is independent of this distance. In many mobile ground based wireless networks, the channel is well modeled by the product of a distance dependent path loss and distance independent shadowing and Rayleigh fading components. The edge probability p in practice is determined by factors such as the terrain, buildings, and vehicles in the area, mobile speed, and coding and interleaving used. In contrast, in airborne networks p may be close to 1.

If 2 nodes are within the distance threshold of each other, then a link between them forms with probability p, independently of other links. In this case, we express the distance threshold as

$$r_{N,p} \sim C^{1/d} \left(\frac{\ln N}{pN}\right)^{1/d}. \quad (7)$$

For percolation, or a large giant component to form, when all potential bonds between all pairs of nodes within the network are equally likely, the known result [8] is

$$p > 1/N. \quad (8)$$

In contrast, the criteria (7) for *connectivity* of every node to the network, when only bonds or edges of length within a distance threshold $r_{N,p}$ can form leads to a different result. In order to derive this result, we first express this distance threshold in terms of the node density n, assuming there are N nodes in a volume of size L^d:

$$r_{n,p} \sim \left[\frac{C}{np}(\ln n + d \ln L)\right]^{1/d} \quad (9)$$

In order to achieve not only connectivity, but also a suitable network capacity, multiple nodes need to be able to transmit simultaneously; in a wireless network this requirement means that to avoid interference $r_{n,p}$ needs to decrease as the node density n grows. Hence the bond probability must satisfy

$$p > \frac{1}{N^\delta} : \quad \delta < 1 \quad (10)$$

Thus the more stringent criteria (10) is necessary in this case, and a simple constant multiple of $1/N$ will not suffice for the bond probability.

One example of how the bond or link probability p can change is illustrated by the approach of a thunderstorm towards people outside communicating in a mobile ad hoc network. The people will then need to take cover close to buildings and then p will decrease to $p' = p \times (1 - f_b)$ due to increased shadowing and blockage, where f_b is the fraction of existing bonds or links that are broken before storm. Following the relocation of the nodes with the onset of the storm, the distance threshold would need to be renormalized to

$$r_{N,p'} = r_{N,p}/(1-f_b)^{1/d}. \quad (11)$$

It is seen from (9) that while changing the network length L can affect the threshold distance, changes in the node density n or bond probability p have a much greater impact.

C. Inhomogeneous Networks

We can generalize to a spatially varying node density that varies slowly compared to r_N. If we assume a single constant distance threshold r_N and bond probability p, then in order to maintain network connectivity, the transmit power could be varied in areas of different node density, as in [6]. Alternatively, we may need to define different r_N in different regimes of node density, according to (9). More generally, the combination of p and n in each local region, determines the local threshold distance that would be needed to achieve connectivity.

D. Relation to Percolation Theory and Network Spanning Cluster

The results of [9] show that for a network spanning cluster to form, the requirement is

$$\sum_{k \geq 1} k(k-2)P(k) \geq 0, \quad (12)$$

where $P(k)$ is the degree distribution of nodes in the network. From (12) it can be shown [9] that this criteria reduces to

$$K > 2, \quad (13)$$

where K is given by (3). Using the parameters we defined in Section III, we express the expected degree of a node in terms of the node density n, the link probability, and the edge distance threshold as

$$< k >= npr_N^d. \quad (14)$$

The criteria (12) for a network spanning cluster to form is less stringent than the criteria (7) for the network to achieve connectivity. For the case of network connectivity depending primarily on the edge threshold distance and the node density, it is shown [7] that the expected node degree grows logarithmically with the number of nodes N. This case network connectivity case is the dense limit of the more general thermodynamic regime. This more general regime requires only that the expected degree of a node is a constant for a network spanning cluster to form, which can be used to set (14) to the desired constant value. Hence in this more general thermodynamic regime formulation, which we

have further generalized to include the bond probability p, a threshold distance can be chosen to adapt to changes in n or p in the network, just as we showed in earlier sections for the connectivity regime.

IV. Implementation of Network Recovery

We consider wireless networks, in which case we assume that the received power P_R, and hence the link data rate R that can be achieved, is related to the distance ρ between nodes as

$$R \sim P_R \sim P_T \rho^{-\alpha}, \tag{15}$$

where the path loss exponent α typically ranges from 2 in free space to values close to 4 for ground to ground propagation. The original data rate R_o in the network is selected according to the path loss equation (15), with $\rho = r_N$, that is according to the maximal edge length, so that all nodes can receive from nodes to which they are connected at the rate R_o.

Following a catastrophic failure and the renormalization of distance proposed in Section III, we now turn to the actual implementation of this renormalization, and hence of network communication, at this new rescaled edge distance threshold. One option is to lower the radios' link data rates according to (15). Alternatively, the same link rate could be maintained if the transmit power of the nodes could be increased according to (15). A third option is that in some cases, a rescaling downward of the carrier frequencies used in transmission would enable the original data rate and transmission powers to be maintained, assuming the spectrum could be made available for this downward shifting of the carrier frequencies. Since the propagation distance scales with the wavelength, the carrier frequencies would need to be scaled down inversely proportionally to the increase in distance threshold.

So if the original data rate was R_o, then after losing a fraction f of nodes randomly, we then could achieve a new rate of only

$$R' \sim R \times \frac{1}{(p \times (1-f))^{-\alpha/d}} \times \left(1 + \frac{\ln(1-f)}{\ln N}\right)^{-\alpha/d}. \tag{16}$$

This new rate can be approximated by the lower bound

$$R' > R(1-f)^{\alpha/d}, \tag{17}$$

if the failures of nodes are independent and identically distributed. If the node failures are correlated, a rate lower than (16) would be expected, as the distance edge threshold would need to be enlarged even more.

If after the fraction f nodes are removed, the same *density* of nodes were desired, then the distance rescaling needed would yield (17). However, the relation (16) indicates that the distance rescaling is actually done to yield an even greater density of nodes when more nodes are added, so as to ensure connectivity. Alternatively, this distance rescaling can be viewed as decreasing the network length L in a finite sized system. So after rescaling according to (17) there would be the same node degree on average, but fewer hops between communicating nodes; hence, the probability of connection would increase.

We present the results of a simple numerical example here, for the case of a very large number of airborne nodes randomly located in space: We assume that before node failures, the distance threshold for a link to form is chosen to be just large enough to ensure connectivity of all nodes to the network. If a random selection of *half the nodes then fail*, then the new distance threshold would need to be roughly 1.3 times that of the original one to ensure connectivity of all nodes. In order to implement this new distance threshold, the data rate on all links could be made roughly 0.6 times smaller. Alternatively, the original data rate on all links could be maintained, if the transmit power of many nodes were increased by about 2 dB. Another alternative for maintaining the original data rate after the failures, while still using the original transmit powers, would be to decrease the carrier frequencies to about 0.8 times their original values. Finally, as a more general alternative, the network could re-establish connectivity through the proposed distance rescaling by some combination of lowering the data rate, increasing the node transmit powers, and decreasing the carrier frequencies to a lesser extent than if only one of these mechanisms is used alone.

In a ground based network or for low flying aircraft, the corresponding changes to the data rate or transmit power that would be needed to re-establish connectivity would be greater than the amounts presented here for an airborne network due to an edge probability $p < 1$. Furthermore, in these situations, particularly when neighboring nodes are already widely separated, there can be limitations on how much distance rescaling may be feasible while maintaining link communication, due to earth curvature and terrain blockage. In this case, there may be a critical fraction of nodes that can fail, above which recovery of network connectivity by link rate reduction, power increases, or carrier frequency decreases will not be feasible.

Another option, for future investigation, is that of simulcast, a modulation by which different users can receive the same signal at different data rates, depending on their locations. Power adjustment may be less advantageous than rate adjustment, as increasing power can cause problems with battery lifetimes, as well as interference to other nodes simultaneously transmitting in the network.

If there were a power shortage or a need to decrease transmit powers, but a desire to achieve the same link rates, equation (15) indicates this situation could be achieved by a *downward rescaling* of the distance threshold. For example if the transmit power needed to be reduced to xP_T where $x < 1$, then the distance threshold would need to be rescaled down by a factor of $x^{1/\alpha}$. If the new rescaled distance threshold is at least as large as (7), then network connectivity can still be maintained at the same link rate. Otherwise, a higher transmit power may be needed, with less of a distance rescaling, and a correspondingly lower link rate, in order to maintain some communications.

V. Summary

We have proposed methods to recover from multiple node failures or power outages that destroy network connectivity. We have proposed starting with an edge threshold distance

rescaling based on a random geometric graph theory model. We have generalized this model and our rescaling to handle the common scenarios in wireless channels which include random aspects, such as fading, shadowing, or blockages. We have derived the critical random bond connectivity for this generalized scenario.

We have suggested potential rescaling of link distances, node transmit powers, carrier frequencies, and *link rates* in order to re-establish *connectivity* across the network. The extension of this work to determine the optimal distance rescaling to maximize network capacity when multiple users transmit simultaneously is a subject for future investigation.

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